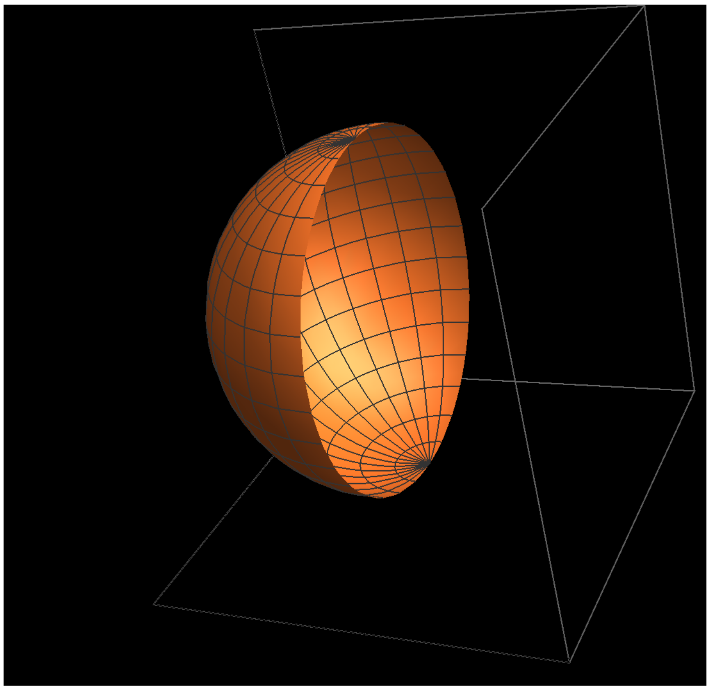
Seja Superfície Parametrizada Regular r: $U \subset \mathbb{R}^2 \to \mathbb{R}^3$

```
In[•]:= x[u_, v_] = Cos[v] Sin[u]; y[u_, v_] = Sin[u] Sin[v];
        z[u_, v_] = Cos[u]; (*Parametrização*)

In[•]:= r[u_, v_] = {x[u, v], y[u, v], z[u, v]} ;

In[•]:= esfera = ParametricPlot3D[r[u, v], {v, -π, 0}, {u, 0, π}, PlotRange → 1.2,
          PlotTheme → "Scientific", Background → Black] (*Gráfico da Superfície*)
```

Out[•]=

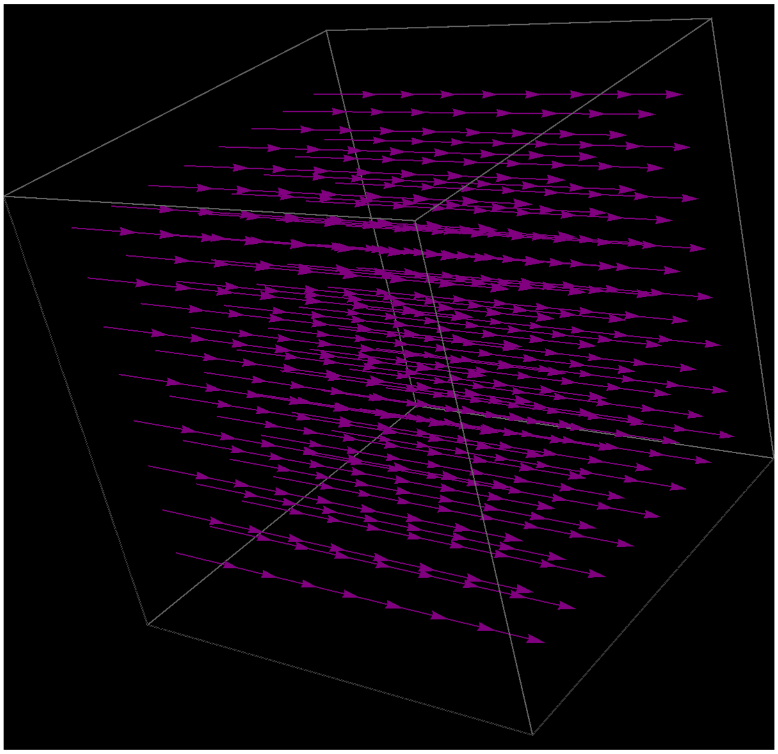


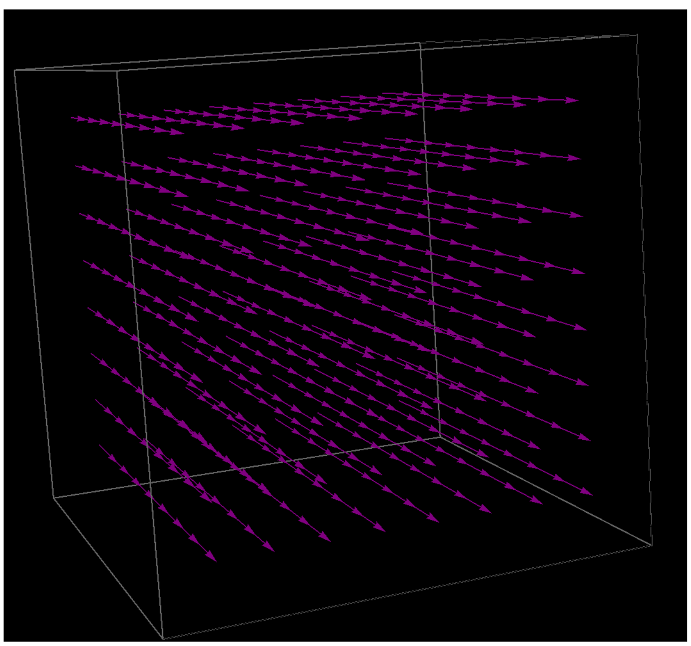
Definamos o campo vetorial ff: $\mathbb{R}^3 \to \mathbb{R}^3$

```
In[•]:= ff = {0, a, 0}; (*Define o campo vetorial constante paralelo ao eixo y*)
```

In[•]:=
```
campo = VectorPlot3D[ff /. a → 4, {x, -1.5, 1.5}, {y, -1.5, 1.5},
  {z, -1.5, 1.5}, VectorStyle → {Arrowheads[0.02], Purple}, Background → Black]
(*Linhas de Campo*)
```

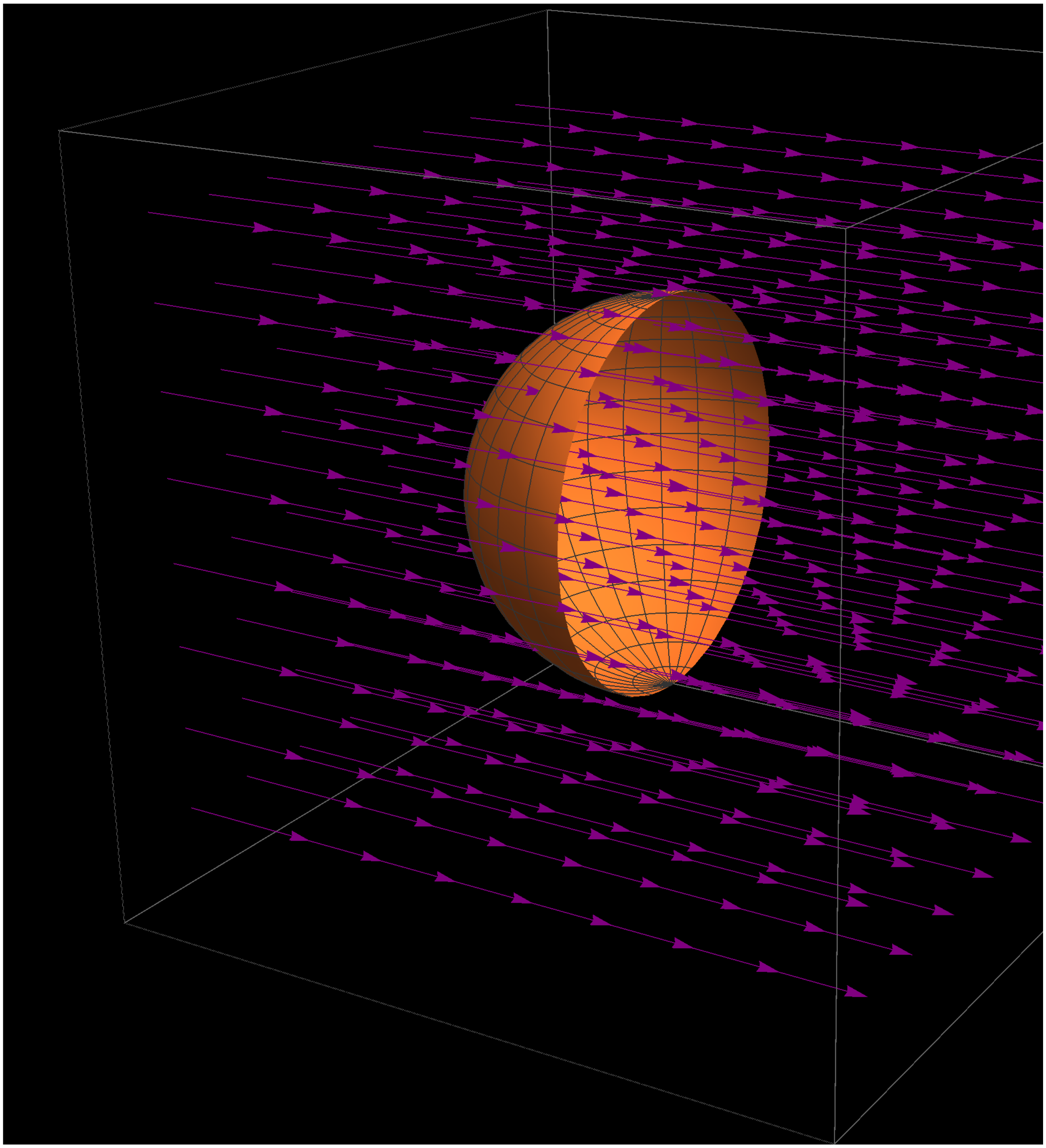
Out[•]=





In[•]:=
```
Show[campo, esfera]
```

Out[•]=



In[•]:=
```
SliceVectorPlot3D[{0, 1, 0}, {x^2 + y^2 + z^2 == 1}, {x, -1, 1},
 {y, -1, 0}, {z, -1, 1}, BoxRatios → {1, 1, 1}, Background → Black]
```

Vetor normal à superfície:

In[•]:=
```
TraditionalForm[MatrixForm[{{i, j, k}, ∂ᵤr[u, v], ∂ᵥr[u, v]}]]
```

$$\begin{pmatrix} i & j & k \\ \cos(u)\cos(v) & \cos(u)\sin(v) & -\sin(u) \\ -\sin(u)\sin(v) & \sin(u)\cos(v) & 0 \end{pmatrix}$$

In[∘]:=
```
r_u = ∂_u r[u, v]; r_v = ∂_v r[u, v];
nn[u, v] = r_u × r_v;  (*Normal - não unitário*)
```

In[∘]:=
```
Simplify[TraditionalForm[nn[u, v]]]
```

Out[∘]//TraditionalForm=

$$\{\sin^2(u)\cos(v),\ \sin^2(u)\sin(v),\ \sin(u)\cos(u)\}$$

In[∘]:=
```
supel = Norm[nn[u, v]];
```

In[∘]:=
```
TraditionalForm[Simplify[supel, Element[{u, v}, Reals]]](*Norma do vetor*)
```

Out[∘]//TraditionalForm=

$$|\sin(u)|$$

Temos, portanto, ds=|sen(u)|du dv.

Note que n= nn/|nn| e ds=|nn|du dv, portanto, n ds=nn du dv

In[∘]:=
```
TraditionalForm[ff.nn[u, v]] (*Cálculo do Produto Escalar*)
```

Out[∘]//TraditionalForm=

$$a\sin^2(u)\sin(v)$$

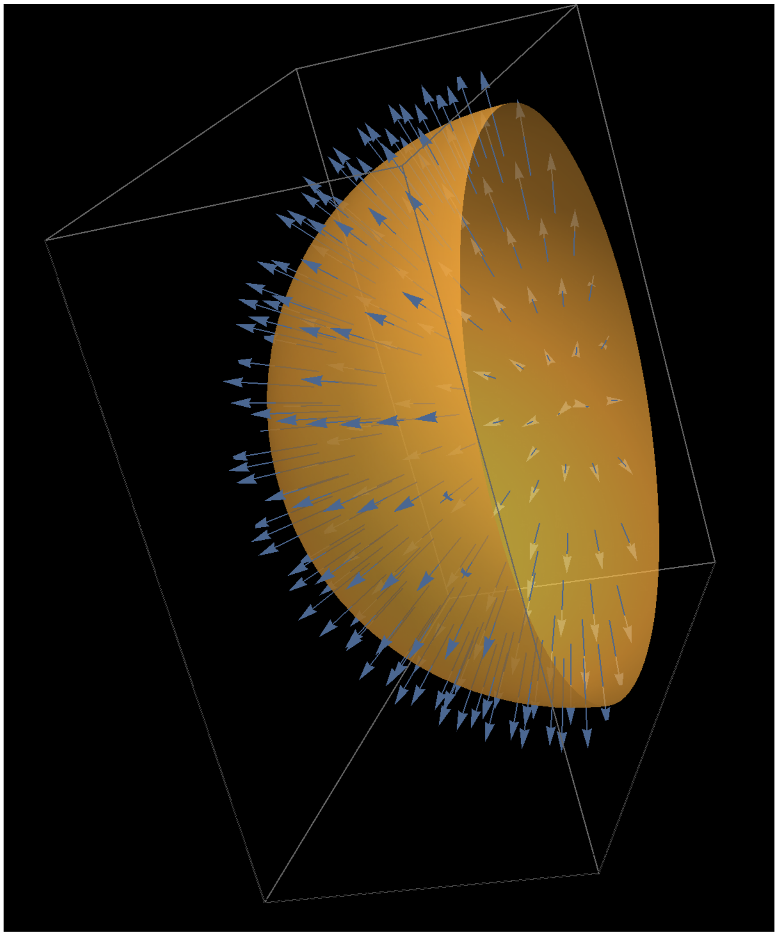
In[∘]:=

$$\int_{-\pi}^{0}\int_{0}^{\pi} \text{ff.nn}[u, v]\, du\, dv$$

Out[∘]= $-a\,\pi$

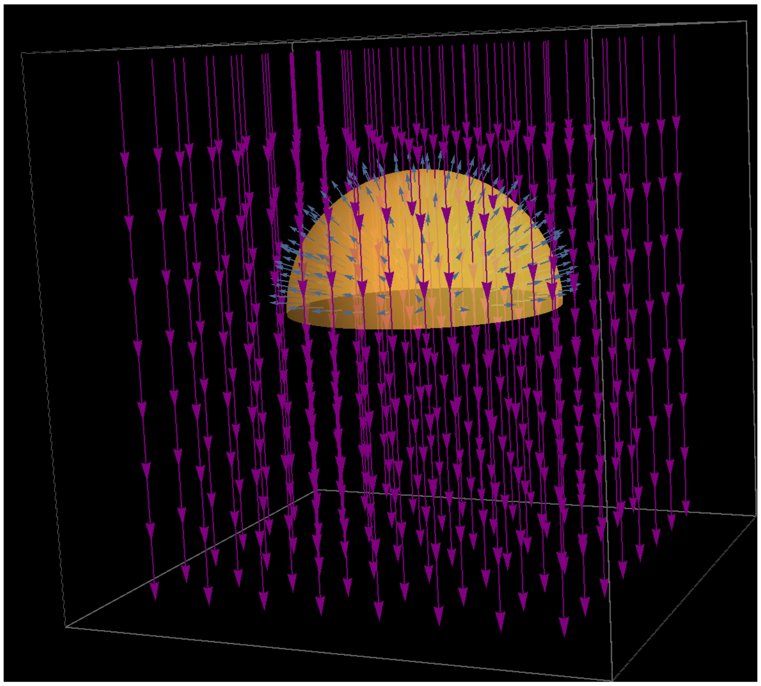
In[∘]:=
```
normal = SliceVectorPlot3D[{x, y, z}, {x^2 + y^2 + z^2 == 1},
  {x, -1, 1}, {y, -1, 0}, {z, -1, 1}, BoxRatios → {1, 1/2, 1},
  VectorPoints → 8, VectorStyle → {Arrowheads[0.02]},
  Background → Black] (*Vetor Normal à superfície*)
```

Out[∘]=

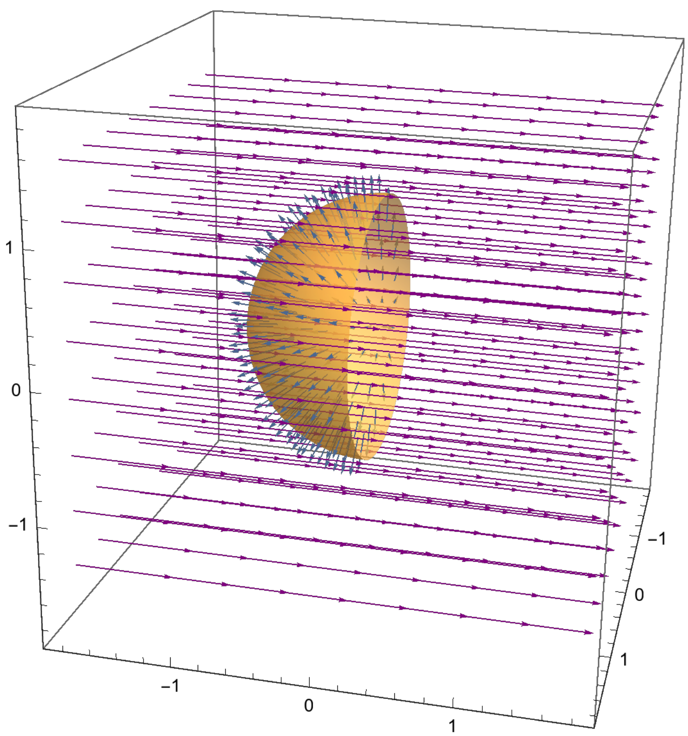


In[•]:= Show[campo, normal]

Out[•]=

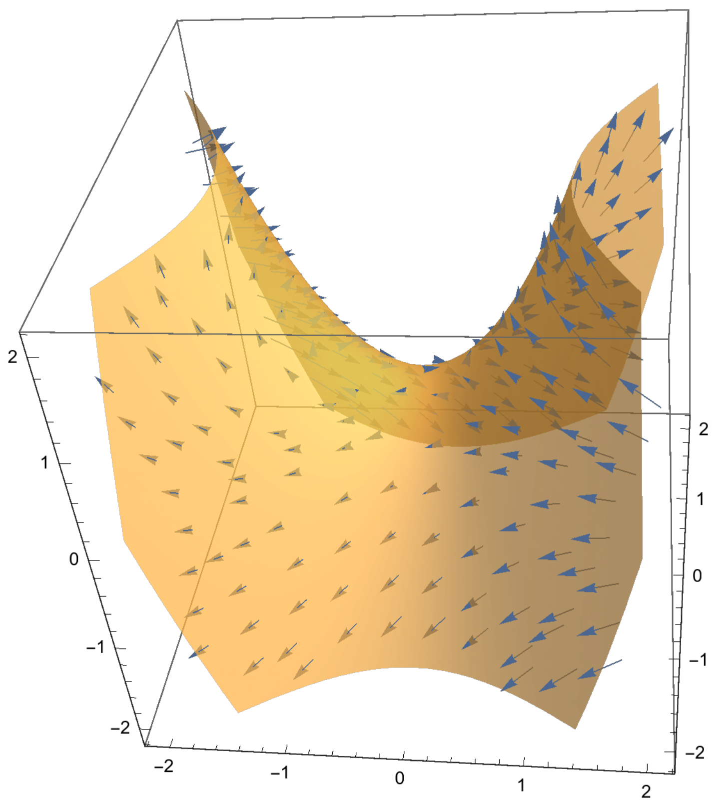


Out[•]=



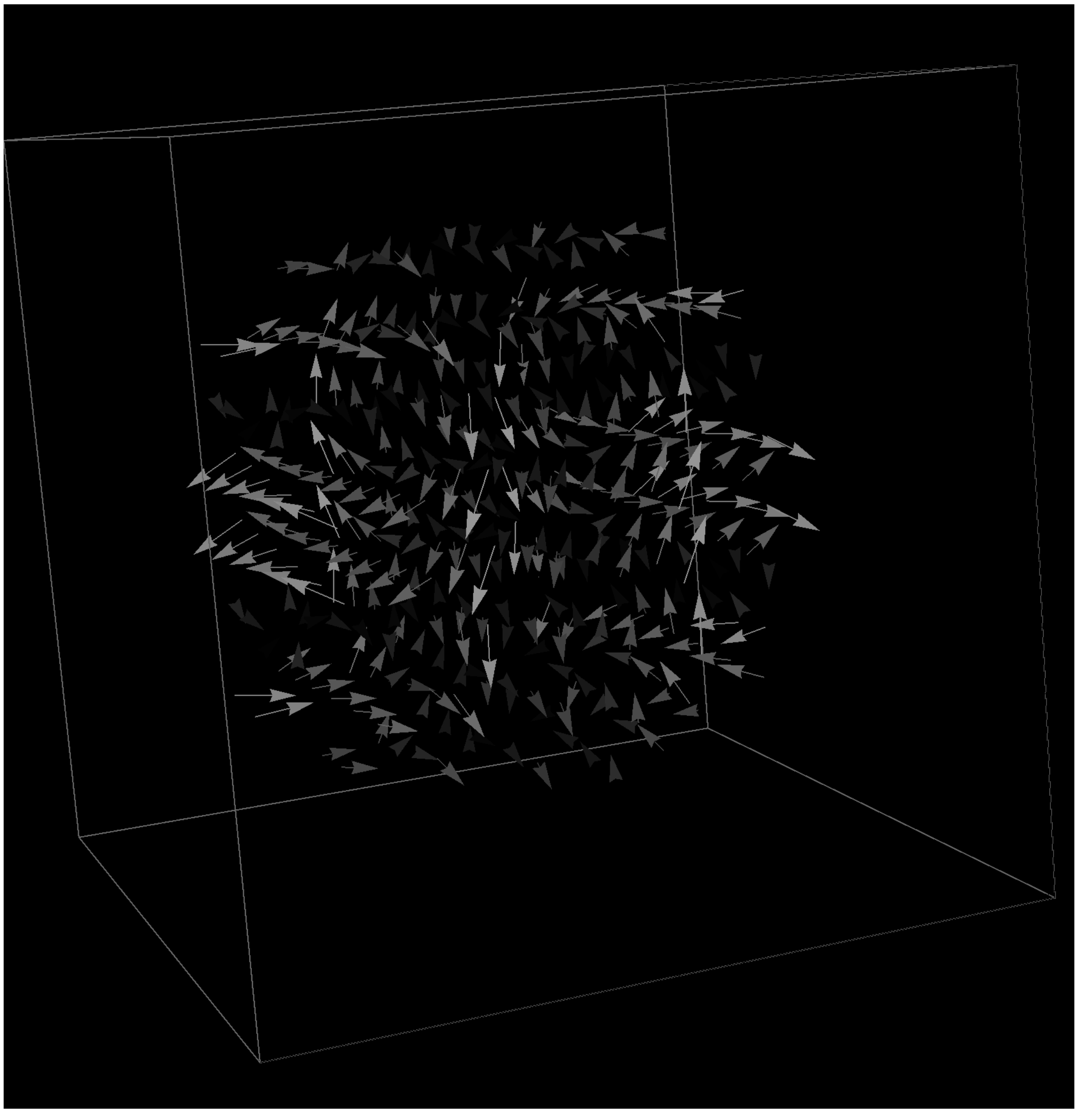
```
SliceVectorPlot3D[{y, -x, x + z}, {x^2 - y^2 - z == 0}, {x, -2, 2},
 {y, -2, 2}, {z, -2, 2}, VectorStyle → Arrowheads[0.02], Background → Black]
```

Out[•]=

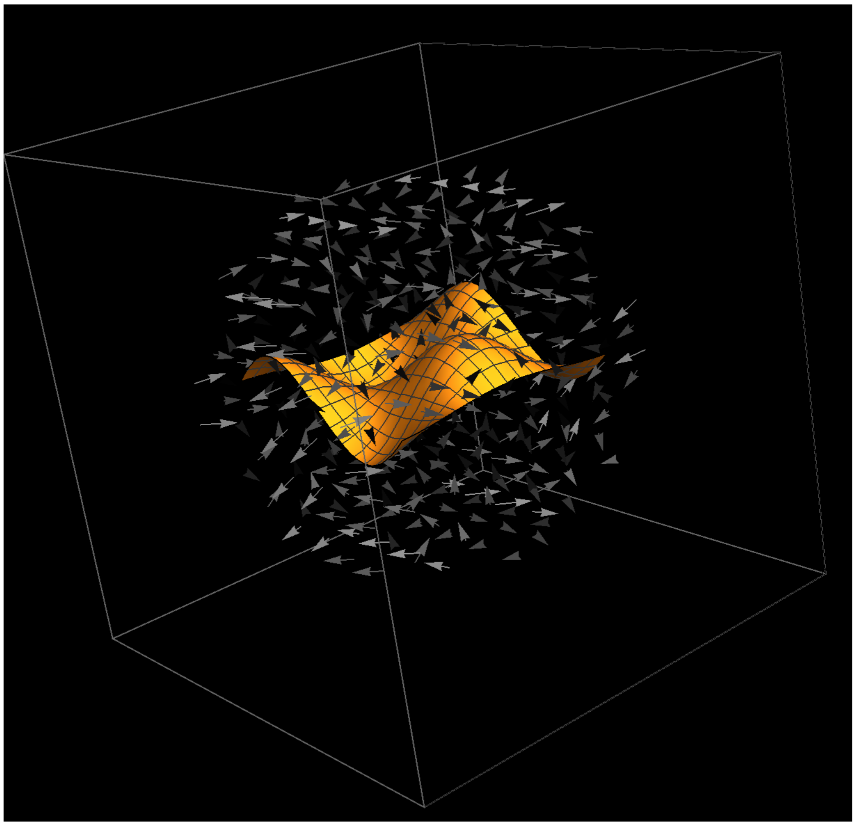


In[•]:=
```
gr = VectorPlot3D[{x Cos[y], z Sin[x], Cos[x + z]}, {x, -5, 5},
  {y, -5, 5}, {z, -5, 5}, VectorPoints → 10, VectorStyle → Arrowheads[0.02],
  VectorScale → {0.1, Scaled[0.5]}, VectorColorFunction → GrayLevel,
  RegionFunction → Function[{x, y, z}, x^2 + y^2 + z^2 <= 5^2], Background → Black]
```

Out[•]=



In[•]:= Show[gr, fig]

Out[•]=


In[•]:= fig = Plot3D[Cos[x] * Sin[y], {x, -3, 3}, {y, -3, 3}]

Out[•]=